Algorithmics of Matching Under Preferences∗

David F. Manlove

School of Computing Science, University of Glasgow, Glasgow G12 8QQ, UK.
david.manlove@glasgow.ac.uk

Errata†

All page and line numbers are given with respect to the published (hard-copy) book.

Frontmatter
• Page ii, line 6: “Iwana” → “Iwama”. (Due to Sofiat Olaosebikan.)

Preface
• Page vii, line -1 to Page viii, line 1: “polynomial-time algorithms)” → “polynomial-time) algorithms”.

Foreword
• Page xiii, line -16: “became” → “become”.

Chapter 1
• Page 5, line 8: “set pairs” → “set of pairs”. (Due to Mechthild Opperud.)
• Page 6, line 3: add “who are not indifferent between the two matchings” after “preferred by the majority of the applicants”.
• Page 17, line 13: “constrast” → “contrast”. (Due to Mechthild Opperud.)
• Page 21, line 17: “attributes” → “attributed”.
• Page 21, lines -4 to -3: change “pairs in which either (i) \( r_i \) is unassigned if she is unassigned in both \( M \) and \( M' \), or (ii)” to “pairs obtained as follows: for each resident \( r_i \), \( r_i \) is unassigned if she is unassigned in both \( M \) and \( M' \), otherwise”.
• Page 21, line -1: “join” → “meet”. (Due to Didac Busquets.)
• Page 22, line 1: “meet” → “join”. (Due to Didac Busquets.)
• Page 23, line 4: add “In an \( sm \) instance, any matching is automatically assumed to have size \( n \).”
• Page 33, line -18: “in an” → “is an”. (Due to Mechthild Opperud.)

†Last modified 29 August 2019.
• Page 35, line 9: “pairs” → “distinct pairs”. (Due to Mechthild Opperud.)

• Page 35, line 12: “all successors of $a_{i_{k-1}}$ from the list of $a_{j_k}$,” → “all successors $a_r$ of $a_{i_{k-1}}$ from the list of $a_{j_k}$, and deleting $a_{j_k}$ from the list of $a_r$.”. (Due to Mechthild Opperud.)

• Page 39, line -8: add “Let $A_M$ denote the set of applicants who are assigned in $M$.” to the end of this paragraph.

• Page 41, lines -7 to -6: add ‘who are not indifferent between the two matchings” after “preferred by the majority of the applicants”.

Chapter 2

• Page 55, line 7: “admits least” → “admits at least”. (Due to Radoslaw Cymer and Mechthild Opperud.)

• Page 55, line 17: “Gusfield and Irving [261]” → “Irving and Leather [319]”.

• Page 56, line 5: “$(M(w), M(m))$” → “$(M(w), M(m))$”. (Due to Ciaran McCreesh.)

• Page 57, line 7: “$D_{I_k}$” → “$D_{I_k}$”.

• Page 57, line -13: “$D_I$” → “$D_{I_k}$”. (Due to Mechthild Opperud.)

• Page 59, line -13: “for the finding” → “for finding”. (Due to Mechthild Opperud.)

• Page 61, line -5: “sex-equality measure measures” → “sex-equality measures”. (Due to Mechthild Opperud.)

• Page 66, line 15: “network stability” → “network stability”. (Due to Radoslaw Cymer, Shuichi Miyazaki and Mechthild Opperud.)

• Page 79, line -15: “by the a” → “by a”. (Due to Mechthild Opperud.)

• Page 80, line -7: “paramters” → “parameters”. (Due to Mechthild Opperud.)

• Page 85, line -3: “$(U \cup W) \setminus S$” → “$(U \cup W) \setminus S$”. (Due to Shuichi Miyazaki and Mechthild Opperud.)

• Page 98, line 1: “median stable matching in” → “median of”. (Due to Mechthild Opperud.)

• Page 98, line 2: “median stable matchings in” → “medians of”. (Due to Mechthild Opperud.)

• Page 99, line -16: “the the” → “the”.

• Page 113, line -2: “$G$ in $n$-choosable” → “$G$ is $n$-choosable”. (Due to Radoslaw Cymer.)

• Page 114, line 4: “the the line graph” → “that the line graph”. (Due to Mechthild Opperud.)
Chapter 3

- Page 138, Algorithm 3.1, add “Require: SMTI instance $I$, “Ensure: return a weakly stable matching $M$ in $I$ such that $|M| \geq \frac{2}{3}s^+(I)$”.
- Page 139, Algorithm 3.2, under line “Require:”, add “Ensure: $w_j$ rejects $m_i$”.
- Page 147, line -10: “prefers $r_i$ to $r_k$” → “prefers $r_k$ to $r_i$”. (Due to Mechthild Opperud.)
- Page 149, line 2: delete “Pareto”.
- Page 149, line 3: “resident-Pareto” → “resident-optimal weakly”.
- Page 149, line 4: “resident-Pareto” → “resident-optimal”.
- Page 149, line 5: “matching $M$” → “weakly stable matching $M$”.
- Page 149, line 12: “resident-Pareto stable” → “resident-optimal”.
- Page 149, line 14: “resident-Pareto” → “resident-optimal weakly”.
- Page 149, line 20: “resident-Pareto” → “resident-optimal weakly”.
- Page 149, line 22: “resident-Pareto” → “resident-optimal”.
- Page 149, line 25: “resident-Pareto” → “resident-optimal weakly”.
- Page 149, line -7: add “Note that an instance of SM may not admit a stable matching that is Pareto optimal for the men – see Sec. 5.7.3.”
- Page 158, line -6: “fewest” → “minimum”.
- Page 160, line -14: “super-stable in every” → “stable in every”. (Due to Mechthild Opperud.)

Chapter 4

- References to the Tan-Hsueh algorithm should be in the index.
- Similarly all references to the Roth-Vande Vate algorithm should be in the index (note that the term “Roth-Vande Vate Mechanism” is used in Chapter 2).
- Page 186, line 7: “exit conditions loop” → “exit conditions”. (Due to Mechthild Opperud.)
- Page 192, line -4: “given worked” → “worked”. (Due to Mechthild Opperud.)
- Page 197, line 5: “inSec.” → “in Sec.” (Due to Mechthild Opperud.)
- Page 200, line -4: “$s + (I)$” → “$s^+(I)$”. (Due to Mechthild Opperud.)
- Page 216, line -4: “an many-many extension” → “a many-many extension”. (Due to Mechthild Opperud.)
Chapter 5

• Page 246, Definition 5.13: the first sentence of Case (3) should read “it involves a couple \((r_i, r_j) \in R_C\) and a pair of (not necessarily distinct) hospitals \(h_k, h_l \in H\) such that \(h_k \neq M(r_i)\), \(h_l \neq M(r_j)\), \((r_i, r_j)\) finds \((h_k, h_l)\) acceptable, and either \((r_i, r_j)\) is unmatched or prefers \((h_k, h_l)\) to \((M(r_i), M(r_j))\), and either”.

• Page 248, Theorem 5.15: “every distinct pair of hospitals” → “every ordered pair of distinct hospitals”.

• Page 251, line -10: “In fact, consistent preference lists need not be responsive” → “In fact, responsive preference lists need not be consistent”. (Due to Mechthild Opperud.)

• Page 254, caption of Figure 5.8: “HRIC” → “HRS”.

• Page 255, line -15: “all possible” → “acceptable”.

• Page 255, line -14: “the each” → “each”.

• Page 255, line -10: “in general exponential” → “in the worst case exponential in”.

• Page 259, line 8: “[498]” → “[499]”.

• Page 259, line 21, “mentioned” → “mentioned”.

• Page 264, line 18: “linear orders gives rise” → “linear orders give rise”. (Due to Mechthild Opperud.)

• Page 264, line 21: “Algorithm spa-s-student” → “Algorithm spa-s-student”. (Due to Mechthild Opperud.)

• Page 268, line 9: “generalistions” → “generalisations”. (Due to Sofiat Olaosebikan.)

• Page 277, line 8: “\(deg_M(t)\)” → “\(deg_M(t)\)” (Due to Radoslaw Cymer.)

• Page 278, line -12: after “Boros et al.”, cite Ref. [109]. (Due to Radoslaw Cymer.)

• Page 286, line -5: “prefers \(\{a_p, a_q\}\) and \(\{a_r, a_s\}\)” → “prefers \(\{a_p, a_q\}\) to \(\{a_r, a_s\}\)”. (Due to Mechthild Opperud.)

• Pages 294-295: Algorithm 5.2, as it stands, may not produce a bistable matching. Instead of line 11, we should delete the pair \((m_k, w_l)\) only if it belongs to \(M\), otherwise the pair should be marked as ineligible (all man–woman pairs are initially eligible). If a man \(m_i\) proposes to a woman \(w_j\) where \((m_i, w_j)\) is marked as ineligible, the procedure is as per lines 4-6 and 8-13 of Algorithm 5.2 (subject to the modifications to line 11 as described), but following any deletions and pairs being marked as ineligible, the pair \((m_i, w_j)\) is not added to \(M\) but is instead deleted. This is as described in [585, Section 5]. (Due to Shuichi Miyazaki and Kazuya Okamoto.)

• Page 295, line 2: “instnance” → “instance”. (Due to Radoslaw Cymer.)
Chapter 6

- Page 311, lines -6 to -2: delete these lines as it is not true in general that $p^-(I) = \beta^-(G)$. However it is true that $p^-(I) \geq \beta^-(G)$ and $p^+(I) = \beta^+(G)$. (Due to Mechthild Opperud.)

- Page 312, lines 1-4: Theorem 6.6 should reference [18]. The second sentence in the theorem statement should be replaced by “The result holds even if each applicant finds at three houses acceptable.” (Due to Mechthild Opperud.)

- Page 312, lines 5-6: “$p^-(I) = \beta^-(G)$” $\rightarrow$ “$p^-(I) \geq \beta^-(G)$”. (Due to Mechthild Opperud.)

- Page 312, lines 11-15: delete from “One way of proving this” up to the end of the paragraph, and replace with “A similar result holds for matchings in a graph: that is, a given graph $G$ admits a maximal matching of size $k$, for each $k$ such that $\beta^-(G) \leq k \leq \beta^+(G)$ [276].” (Due to Mechthild Opperud.)

- Page 313, line -13: insert “in” after “better off”.

- Page 315, line 10: the case where $r = 1$ should be dealt with separately. In this case, each of $a_{i_0}$ and $h_1$ is unassigned, and $h_1 \in A(a_{i_0})$. (Due to Baharak Rastegari.)

- Page 315, lines 24-27: replace by the following. Given an improving coalition $C$, let $M'$ be the matching

$$M' = (M \setminus \{(a_{i_j}, M(a_{i_j})) : 1 \leq j \leq r - 1\}) \cup \{(a_{i_j}, M(a_{i_{j+1}})) : 0 \leq j \leq r - 2\}.$$ 

Then $M''$ is defined to be the matching obtained from $M$ by satisfying $C$, where $M'' = (M' \setminus \{(a_{i_0}, M(a_{i_0}))\}) \cup \{(a_{i_{r-1}}, h_k)\}$ in the case of an alternating path coalition, $M'' = M' \cup \{(a_{i_{r-1}}, h_k)\}$ in the case of an augmenting path coalition and $M'' = (M' \setminus \{(a_{i_0}, M(a_{i_0}))\}) \cup \{(a_{i_{r-1}}, M(a_{i_0}))\}$ in the case of a cyclic coalition. (Due to Baharak Rastegari.)

- Page 317: the statement prior to Proposition 6.14 is incorrect. It is open as to whether the time complexity stated in Proposition 6.14 is true. However note that in an instance $I$ of Hat in which every applicant’s preference list comprises a single tie, the Pareto optimal matchings in $I$ are precisely the maximum matchings in the underlying graph $G$. Thus an $O(m)$ algorithm for finding a Pareto optimal matching in $I$ would imply an $O(m)$ algorithm to find a maximum matching in an arbitrary bipartite graph. (Due to Baharak Rastegari.)

- Page 320, lines 2-4: the sentence beginning “Also $M$ is trade-in-free” should read “Also $M$ is trade-in-free” if there is no applicant–house pair $(a_i, h_j)$ such that $a_i$ is assigned in $M$, $h_j$ is undersubscribed in $M$ and $a_i$ prefers $h_j$ to $M(a_i)$.” (Due to Andre Veski.)

- Page 321, lines 3 and 9 of Algorithm 6.3: $A$ should be $A_M$. (Due to Zhiyuan Lin.)

- Page 322, after line 7 of Algorithm 6.4: add “if ($Q \neq \emptyset$) then remove $head(Q)$ from $L_k$” – this is to prevent $a_i = head(Q)$ having $h_k$ removed from its list, because $a_i$ will be promoted to $h_k$ at the next iteration of the while loop. (Due to Zhiyuan Lin.)

- Page 323, line 19: “who $a_j$ envies” $\rightarrow$ “whom $a_j$ envies”. (Due to Mechthild Opperud.)

- Page 325, line 16: “maximum” $\rightarrow$ “maximum”. (Due to Agnes Cseh.)
Chapter 7

- Page 339, line 10: delete “indexsolvabilityprobability”. (Due to Shuichi Miyazaki and Mechthild Opperud.)
- Page 339, line 10: replace “indexsolvability probability” by the corresponding invisible \LaTeX command. (Due to Mechthild Opperud.)
- Page 339, lines 13 and 15: “proportion” → “percentage”. (Due to Mechthild Opperud.)
- Page 339, line 14: “1000” → “100%” and “556” → “55.6%”. (Due to Mechthild Opperud.)
- Page 339, line 16: “1000” → “100%” and “2” → “0.2%”. (Due to Mechthild Opperud.)
- Page 343, line 15: \( s(T_i) \) → \( |s(T_i)| \). (Due to Shuichi Miyazaki.)
- Page 355, line 9: add the following text after “a contradiction”: “Similarly if a house \( h_j \) ∈ \( H \) is unassigned in \( M \), let \( a_i \) be any applicant such that \( h_j \) ∈ \( f(a_i) \). If \( a_i \) is unassigned in \( M \), clearly \( M \cup \{(a_i, h_j)\} \) is more popular than \( M \), a contradiction. Hence let \( h_k = M(a_i) \). Then \( M' = (M \setminus \{(a_i, h_k)\}) \cup \{(a_i, h_j)\} \) satisfies \( |P(M', M)| = |P(M, M')| = 0 \), a contradiction.” (Due to Mechthild Opperud.)
- Page 356, line -11: after “majority consensus” add “(among the applicants who are not indifferent between \( M \) and \( M' \)).”
- Page 357, line -16: after “majority of the applicants” add “(who are not indifferent).”
- Page 361, line 4: “to case that” → “to the case that” (Due to Mechthild Opperud.)
- Page 366, line -1: after “weighted majority of the applicants” add “who are not indifferent between the two matchings”.
- Page 368, line 12: after “majority of the agents” add “who are not indifferent between the two matchings”.
- Page 378, line 5: after “majority of the agents” add “who are not indifferent between the two matchings”.
- Page 380, lines -8 to -7: “each of the problems of finding a popular matching and a maximum popular matching in the context of srti and smti” → “the problem of finding a popular matching or reporting that none exists in the context of srti or smti”

Chapter 8

- Page 400, line 11: add “Note that the components in the profile of an alternating path can be negative, which is not true in the case of the profile of a matching.”
- Page 401, Algorithm 8.3, line 3, 25 and 26: “\( O_r \)” → “\( O_r^- \), where \( O_r^- = (p_1^-, \ldots, p_r^-) \), \( p_1^- = -n_1 - 1 \) and \( p_k^- = 0 \) (2 ≤ \( k \) ≤ \( r \)). (Due to Augustine Kwanashie.)
- Page 402, line 16: “\( O_r \)” → “\( O_r^- \), where \( O_r^- = (p_1^-, \ldots, p_r^-) \), \( p_1^- = -n_1 - 1 \) and \( p_k^- = 0 \) (2 ≤ \( k \) ≤ \( r \)),.” (Due to Augustine Kwanashie.)
• Page 404, line 8: “1 ≤ s < β” → “1 ≤ s ≤ β”. (Due to Mechthild Opperud.)

• Page 404, lines -9 and -8: “Let \( O_r' \) be the vector \( \langle p_1, \ldots, p_r \rangle \), where \( p_k = 0 \) (1 ≤ k ≤ r − 1) and \( p_r = n + 1 \).” → “Let \( O_r^+ \) be the vector \( \langle p_1^+, \ldots, p_r^+ \rangle \), where \( p_k^+ = 0 \) (1 ≤ k ≤ r − 1) and \( p_r^+ = n_1 + 1 \).”

• Page 404, lines -8, -3: \( O_r' \) → \( O_r^+ \).

• Page 409, line -17: “such each paper” → “such that each paper”. (Due to Mechthild Opperud.)

Bibliography

• Page 419, reference 35: “Exchance-proofness” → “Exchange-proofness”. (Due to Mechthild Opperud.)

• Page 420, reference 50: In the order of authors, the order of Mitchell and Okamoto should be swapped.

• Page 426, reference 129: “How hard is to find” → “How hard is it to find”. (Due to Mechthild Opperud.)

• Page 430, reference 198: “Sjostrand” → “Sjöstrand”. (Due to Radoslaw Cymer.)

• Page 433, reference 238: “Maximale systeme unabhängig kanten” → “Maximale Systeme unabhängiger Kanten”. (Due to Radoslaw Cymer and Mechthild Opperud.) Also “1965” → “1964”.

• Page 436, reference 272: The title should read “Improved approximation results for the stable marriage problem.”


• Page 440, reference 330: The full title is “The stable fixtures problem – A many-to-many extension of stable roommates”. (Due to Radoslaw Cymer.)

• Page 445, reference 394: The full title is “Mariages stables et leurs relations avec d’autres problèmes combinatoires”. (Due to Mechthild Opperud.)

• Page 451, reference 462: “\( o(n^3 \log n) \)” → “\( O(n^3 \log n) \)”. (Due to Radoslaw Cymer.)

• Page 452, reference 476: “Die theorie der regulären graphs” → “Die Theorie der regulären Graphs”. (Due to Radoslaw Cymer.)

• Page 457, reference 556: “29” → “30”. (Due to Radoslaw Cymer.)

• Page 458, reference 578: “Tallin” → “Tallinn”. (Due to Radoslaw Cymer.)

• Page 459, reference 590: “18, 1” → “38, 3”. (Due to Radoslaw Cymer.)

Glossary

• Page 461, line -17: add notation for \( A_M \), the applicants who are assigned in \( M \) (the context is HA).

Index

• Page 488, column 2, line 20: “FRee” → “Free”. (Due to Ágnes Cseh.)