Algorithmics of Matching Under Preferences

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Errata†

All page and line numbers are given with respect to the published (hard-copy) book.

Frontmatter

• Page ii, line 6: “Iwana” → “Iwama”. (Due to Sofiat Olaosebikan.)

Preface

• Page vii, line -1 to Page viii, line 1: “polynomial-time algorithms)” → “polynomial-time) algorithms”.

Foreword

• Page xiii, line -16: “became” → “become”.

Chapter 1

• Page 5, line 8: “set pairs” → “set of pairs”. (Due to Mechthild Opperud.)
• Page 6, line 3: add “who are not indifferent between the two matchings” after “preferred by the majority of the applicants”.
• Page 17, line 13: “constrast” → “contrast”. (Due to Mechthild Opperud.)
• Page 21, line 17: “attributes” → “attributed”.
• Page 21, lines -4 to -3: change “pairs in which either (i) $r_i$ is unassigned if she is unassigned in both $M$ and $M'$, or (ii)” to “pairs obtained as follows: for each resident $r_i$, $r_i$ is unassigned if she is unassigned in both $M$ and $M'$, otherwise”.
• Page 21, line -1: “join” → “meet”. (Due to Didac Busquets.)
• Page 22, line 1: “meet” → “join”. (Due to Didac Busquets.)
• Page 23, line 4: add “In an SM instance, any matching is automatically assumed to have size $n$.”
• Page 33, line -18: “in an” → “is an”. (Due to Mechthild Opperud.)

†Last modified 12 July 2017.
• Page 35, line 9: “pairs” → “distinct pairs”. (Due to Mechthild Opperud.)

• Page 35, line 12: “all successors of $a_{ik-1}$ from the list of $a_{jk}$,” → “all successors $a_r$ of $a_{ik-1}$ from the list of $a_{jk}$, and deleting $a_{jk}$ from the list of $a_r$.”. (Due to Mechthild Opperud.)

• Page 39, line -8: add “Let $A_M$ denote the set of applicants who are assigned in $M$.” to the end of this paragraph.

• Page 41, lines -7 to -6: add ‘who are not indifferent between the two matchings” after “preferred by the majority of the applicants”.

Chapter 2

• Page 55, line 7: “admits least” → “admits at least”. (Due to Radoslaw Cymer and Mechthild Opperud.)

• Page 55, line 17: “Gusfield and Irving [261]” → “Irving and Leather [319]”.

• Page 57, line -13: “$D_I$” → “$D_{I_k}$”. (Due to Mechthild Opperud.)

• Page 59, line -13: “for the finding” → “for finding”. (Due to Mechthild Opperud.)

• Page 61, line -5: “sex-equality measure measures” → “sex-equality measures”. (Due to Mechthild Opperud.)

• Page 66, line 15: “network!stability” → “network stability”. (Due to Radoslaw Cymer, Shuichi Miyazaki and Mechthild Opperud.)

• Page 79, line -15: “by the a” → “by a”. (Due to Mechthild Opperud.)

• Page 80, line -7: “paramters” → “parameters”. (Due to Mechthild Opperud.)

• Page 85, line -3: “$(U \cup W) \setminus S$” → “($U \cup W) \setminus S$”. (Due to Shuichi Miyazaki and Mechthild Opperud.)

• Page 98, line 1: “median stable matching in” → “median of”. (Due to Mechthild Opperud.)

• Page 98, line 2: “median stable matchings in” → “medians of”. (Due to Mechthild Opperud.)

• Page 99, line -16: “the the” → “the”.

• Page 113, line -2: “$G$ in n-choosable” → “$G$ is n-choosable”. (Due to Radoslaw Cymer.)

• Page 114, line 4: “the the line graph” → “that the line graph”. (Due to Mechthild Opperud.)

Chapter 3

• Page 138, Algorithm 3.1, add “Require: SMTI instance $I$”, “Ensure: return a weakly stable matching $M$ in $I$ such that |$M$| $\geq \frac{2}{3}s^+(I)$”.

• Page 139, Algorithm 3.2, under line “Require:”, add “Ensure: $w_j$ rejects $m_i$”. 

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• Page 147, line -10: “prefers $r_i$ to $r_k$” $\rightarrow$ “prefers $r_k$ to $r_i$”. (Due to Mechthild Opperud.)

• Page 149, line 2: delete “Pareto”.

• Page 149, line 3: “resident-Pareto” $\rightarrow$ “resident-optimal weakly”.

• Page 149, line 4: “resident-Pareto” $\rightarrow$ “resident-optimal”.

• Page 149, line 5: “matching $M'$” $\rightarrow$ “weakly stable matching $M'$”.

• Page 149, line 12: “resident-Pareto stable” $\rightarrow$ “resident-optimal”.

• Page 149, line 14: “resident-Pareto” $\rightarrow$ “resident-optimal weakly”.

• Page 149, line 20: “resident-Pareto” $\rightarrow$ “resident-optimal weakly”.

• Page 149, line 22: “resident-Pareto” $\rightarrow$ “resident-optimal”.

• Page 149, line 25: “resident-Pareto” $\rightarrow$ “resident-optimal weakly”.

• Page 149, line -7: add “Note that an instance of sm may not admit a stable matching that is Pareto optimal for the men – see Sec. 5.7.3.”

• Page 158, line -6: “fewest” $\rightarrow$ “minimum”.

• Page 160, line -14: “super-stable in every” $\rightarrow$ “stable in every”. (Due to Mechthild Opperud.)

Chapter 4

• References to the Tan-Hsueh algorithm should be in the index.

• Similarly all references to the Roth–Vande Vate algorithm should be in the index (note that the term “Roth–Vande Vate Mechanism” is used in Chapter 2).

• Page 186, line 7: “exit conditions loop” $\rightarrow$ “exit conditions”. (Due to Mechthild Opperud.)

• Page 192, line -4: “given worked” $\rightarrow$ “worked”. (Due to Mechthild Opperud.)

• Page 197, line 5: “inSec.” $\rightarrow$ “in Sec.” (Due to Mechthild Opperud.)

• Page 200, line -4: “$s + (I)$” $\rightarrow$ “$s^+(I)$”. (Due to Mechthild Opperud.)

• Page 216, line -4: “an many-many extension” $\rightarrow$ “a many-many extension”. (Due to Mechthild Opperud.)

Chapter 5

• Page 246, Definition 5.13: the first sentence of Case (3) should read “it involves a couple $(r_i, r_j) \in R_C$ and a pair of (not necessarily distinct) hospitals $h_k, h_l \in H$ such that $h_k \neq M(r_i), h_l \neq M(r_j)$, $(r_i, r_j)$ finds $(h_k, h_l)$ acceptable, and either $(r_i, r_j)$ is unmatched or prefers $(h_k, h_l)$ to $(M(r_i), M(r_j))$, and either”.

• Page 248, Theorem 5.15: “every distinct pair of hospitals” $\rightarrow$ “every ordered pair of distinct hospitals”.

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• Page 251, line -10: “In fact, consistent preference lists need not be responsive” → “In fact, responsive preference lists need not be consistent”. (Due to Mechthild Opperud.)

• Page 254, caption of Figure 5.8: “HRIC” → “HRS”.

• Page 255, line -15: “all possible” → “acceptable”.

• Page 255, line -14: “the each” → “each”.

• Page 255, line -10: “in general exponential” → “in the worst case exponential in”.

• Page 259, line 8: “[498]” → “[499]”.

• Page 259, line 21, “mentiond” → “mentioned”. (Due to Mechthild Opperud.)

• Page 264, line 18: “linear orders gives rise” → “linear orders give rise”. (Due to Mechthild Opperud.)

• Page 264, line 21: “Algorithm spa-s-student” → “Algorithm spa-s-student”. (Due to Mechthild Opperud.)

• Page 268, line 9: “generalistions” → “generalisations”. (Due to Sofiat Olaosebikan.)

• Page 277, line 8: “degM(t)” → “degM(t)”. (Due to Radoslaw Cymer.)

• Page 278, line -12: after “Boros et al.”, cite Ref. [109]. (Due to Radoslaw Cymer.)

• Page 286, line -5: “prefers \( \{a_p, a_q\} \) and \( \{a_r, a_s\} \)” → “prefers \( \{a_p, a_q\} \) to \( \{a_r, a_s\} \)”. (Due to Mechthild Opperud.)

• Pages 294-295: Algorithm 5.2, as it stands, may not produce a bistable matching. Instead of line 11, we should delete the pair \((m_k, w_l)\) only if it belongs to \(M\), otherwise the pair should be marked as ineligible (all man–woman pairs are initially eligible). If a man \(m_i\) proposes to a woman \(w_j\) where \((m_i, w_j)\) is marked as ineligible, the procedure is as per lines 8-13 of Algorithm 5.2 (subject to the modifications to line 11 as described), but following any deletions and pairs being marked as ineligible, the pair \((m_i, w_j)\) is not added to \(M\) but is instead deleted. This is as described in [585, Section 5]. (Due to Shuichi Miyazaki and Kazuya Okamoto.)

• Page 295, line 2: “instnace” → “instance”. (Due to Radoslaw Cymer.)

Chapter 6

• Page 311, lines -6 to -2: delete these lines as it is not true in general that \(p^-(I) = \beta^-(G)\). However it is true that \(p^-(I) \geq \beta^-(G)\) and \(p^+(I) = \beta^+(G)\). (Due to Mechthild Opperud.)

• Page 312, lines 1-4: Theorem 6.6 should reference [18]. The second sentence in the theorem statement should be replaced by “The result holds even if each applicant finds at three houses acceptable.” (Due to Mechthild Opperud.)

• Page 312, lines 5-6: “\(p^-(I) = \beta^-(G)\)” → “\(p^-(I) \geq \beta^-(G)\)”. (Due to Mechthild Opperud.)
• Page 312, lines 11-15: delete from “One way of proving this” up to the end of the paragraph, and replace with “A similar result holds for matchings in a graph: that is, a given graph $G$ admits a maximal matching of size $k$, for each $k$ such that $\beta^-(G) \leq k \leq \beta^+(G)$ [276].” (Due to Mechthild Opperud.)

• Page 313, line -13: insert “in” after “better off”.

• Page 315, line 10: the case where $r = 1$ should be dealt with separately. In this case, each of $a_{i_0}$ and $h_k$ is unassigned, and $h_k \in A(a_{i_0})$. (Due to Baharak Rastegari.)

• Page 315, lines 24-27: replace by the following. Given an improving coalition $C$, let $M'$ be the matching

$$M' = (M\setminus\{(a_{i_j}, M(a_{i_j})) : 1 \leq j \leq r - 1\}) \cup \{(a_{i_j}, M(a_{i_{j+1}})) : 0 \leq j \leq r - 2\}.$$ 

Then $M''$ is defined to be the matching obtained from $M$ by satisfying $C$, where $M'' = (M\setminus\{(a_{i_0}, M(a_{i_0}))\}) \cup \{(a_{i_{r-1}}, h_k)\}$ in the case of an alternating path coalition, $M'' = M' \cup \{(a_{i_{r-1}}, h_k)\}$ in the case of an augmenting path coalition and $M'' = (M\setminus\{(a_{i_0}, M(a_{i_0}))\}) \cup \{(a_{i_{r-1}}, M(a_{i_0}))\}$ in the case of a cyclic coalition. (Due to Baharak Rastegari.)

• Page 317: the statement prior to Proposition 6.14 is incorrect. It is open as to whether the time complexity stated in Proposition 6.14 is true. However note that in an instance $I$ of hat in which every applicant’s preference list comprises a single tie, the Pareto optimal matchings in $I$ are precisely the maximum matchings in the underlying graph $G$. Thus an $O(m)$ algorithm for finding a Pareto optimal matching in $I$ would imply an $O(m)$ algorithm to find a maximum matching in an arbitrary bipartite graph. (Due to Baharak Rastegari.)

• Page 320, lines 2-4: the sentence beginning “Also $M$ is trade-in-free” should read “Also $M$ is trade-in-free if there is no applicant–house pair $(a_i, h_j)$ such that $a_i$ is assigned in $M$, $h_j$ is undersubscribed in $M$ and $a_i$ prefers $h_j$ to $M(a_i)$.” (Due to Andre Veski.)

• Page 321, lines 3 and 9 of Algorithm 6.3: $A$ should be $A_M$. (Due to Zhiyuan Lin.)

• Page 322, after line 7 of Algorithm 6.4: add “if $(Q \neq \emptyset)$ then remove $head(Q)$ from $L_k$” – this is to prevent $a_t = head(Q)$ having $h_k$ removed from its list, because $a_t$ will be promoted to $h_k$ at the next iteration of the while loop. (Due to Zhiyuan Lin.)

• Page 323, line 19: “who $a_j$ envies” $\rightarrow$ “whom $a_j$ envies”. (Due to Mechthild Opperud.)

• Page 325, line 16: “mxaimum” $\rightarrow$ “maximum”. (Due to Ágnes Cseh.)

Chapter 7

• Page 339, line 10: delete “indexsolvabilityprobability”. (Due to Shuichi Miyazaki and Mechthild Opperud.)

• Page 339, line 10: replace “indexsolvability probability” by the corresponding invisible \texttt{\LaTeX} command. (Due to Mechthild Opperud.)

• Page 339, lines 13 and 15: “proportion” $\rightarrow$ “percentage”. (Due to Mechthild Opperud.)
• Page 339, line 14: “1000” → “100%” and “556” → “55.6%”. (Due to Mechthild Opperud.)

• Page 339, line 16: “1000” → “100%” and “2” → “0.2%”. (Due to Mechthild Opperud.)

• Page 343, line 15: \[ s(T_i) \rightarrow |s(T_i)|. \] (Due to Shuichi Miyazaki.)

• Page 355, line 9: add the following text after “a contradiction”: “Similarly if a house \( h_j \in H \) is unassigned in \( M \), let \( a_i \) be any applicant such that \( h_j \in f(a_i) \). If \( a_i \) is unassigned in \( M \), clearly \( M \cup \{(a_i, h_j)\} \) is more popular than \( M \), a contradiction. Hence let \( h_k = M(a_i) \). Then \( M' = (M \setminus \{(a_i, h_k)\}) \cup \{(a_i, h_j)\} \) satisfies \( |P(M', M)| = |P(M, M')| = 0 \), a contradiction.” (Due to Mechthild Opperud.)

• Page 356, line -11: after “majority consensus” add “(among the applicants who are not indifferent between \( M \) and \( M' \))”.

• Page 357, line -16: after “majority of the applicants” add “(who are not indifferent)”.

• Page 361, line 4: “to case that” → “to the case that” (Due to Mechthild Opperud.)

• Page 366, line -1: after “weighted majority of the applicants” add “who are not indifferent between the two matchings”.

• Page 368, line 12: after “majority of the agents” add “who are not indifferent between the two matchings”.

• Page 378, line 5: after “majority of the agents” add “who are not indifferent between the two matchings”.

• Page 380, lines -9 to -7: “each of the problems of finding a popular matching and a maximum popular matching in the context of SRTI and SMTI” → “the problem of finding a popular matching or reporting that none exists in the context of SRTI or SMTI”

Chapter 8

• Page 400, line 11: add “Note that the components in the profile of an alternating path can be negative, which is not true in the case of the profile of a matching.”

• Page 401, Algorithm 8.3, line 3, 25 and 26: \( O_r \rightarrow O_r^- \). (Due to Augustine Kwanashie.)

• Page 402, line 16: \( O_r^- \rightarrow O_r^- \), where \( O_r^- = (p_1^-, \ldots, p_r^-), p_1^- = -n_1 - 1 \) and \( p_k^- = 0 \) (\( 2 \leq k \leq r \)).” (Due to Augustine Kwanashie.)

• Page 404, line 8: “1 ≤ s < β^*” → “1 ≤ s ≤ β”. (Due to Mechthild Opperud.)

• Page 404, lines -9 and -8: “Let \( O_r' \) be the vector \( \langle p_1, \ldots, p_r \rangle \), where \( p_k = 0 \) (\( 1 \leq k \leq r - 1 \)) and \( p_r = n_1 + 1 \)” → “Let \( O_r^+ \) be the vector \( \langle p_1^+, \ldots, p_r^+ \rangle \), where \( p_k^+ = 0 \) (\( 1 \leq k \leq r - 1 \)) and \( p_r^+ = n_1 + 1 \)”.

• Page 404, lines -8, -3: \( O_r' \rightarrow O_r^+ \).

• Page 409, line -17: “such each paper” → “such that each paper”. (Due to Mechthild Opperud.)
Bibliography

- Page 419, reference 35: “Exchance-proofness” → “Exchange-proofness”. (Due to Mechthild Opperud.)
- Page 426, reference 129: “How hard is to find” → “How hard is it to find”. (Due to Mechthild Opperud.)
- Page 430, reference 198: “Sjostrand” → “Sjöstrand”. (Due to Radoslaw Cymer.)
- Page 433, reference 238: “Maximale systeme unabh¨ angiger kanten” → “Maximale Systeme unabh¨ angeiger Kanten”. (Due to Radoslaw Cymer and Mechthild Opperud.) Also “1965” → “1964”.
- Page 440, reference 330: The full title is “The stable fixtures problem – A many-to-many extension of stable roommates”. (Due to Radoslaw Cymer.)
- Page 445, reference 394: The full title is “Mariages stables et leurs relations avec d’autres problèmes combinatoires”. (Due to Mechthild Opperud.)
- Page 451, reference 462: “o(n³ log n)” → “O(n³ log n)”. (Due to Radoslaw Cymer.)
- Page 452, reference 476: “Die theorie der regulären graphs” → “Die Theorie der regulären Graphs”. (Due to Radoslaw Cymer.)
- Page 457, reference 556: “29” → “30”. (Due to Radoslaw Cymer.)
- Page 458, reference 578: “Tallin” → “Tallinn”. (Due to Radoslaw Cymer.)
- Page 459, reference 590: “18, 1” → “38, 3”. (Due to Radoslaw Cymer.)

Glossary

- Page 461, line -17: add notation for $A_M$, the applicants who are assigned in $M$ (the context is $HA$).

Index

- Page 488, column 2, line 20: “FRee” → “Free”. (Due to Ágnes Cseh.)